Lab 10

Thermodynamics

A. Purpose

To observe the thermal radiation from different surfaces under various heating temperatures, to test Stefan-Boltzmann law, and study gas law by heat engine and the measurement of the heat capacity ratios of different gases.

B. Introduction

Thermodynamics is the study of heat, work, and temperature, and their relation to energy, radiation, and physical properties of matter. As in all science, thermodynamics is based on experimental observation and the findings have been formalized into four basic laws of thermodynamics. This experiment focuses on the observations of thermodynamic systems.

1. Stefan-Boltzmann law

All objects with finite temperature (above absolute zero) radiate. This thermal radiation is an energy transfer via the emission of electromagnetic waves. In 1859, Kirchhoff proposed the law of radiation, pointing out that under a certain temperature, the absorption rate, as well as emission rate of an object to exposed radiation, is related to the surface property of the object. In 1879, Josef Stefan deduced from the experimental results that the radiated energy from an object to the surroundings per second per unit area is proportional to the fourth power of its absolute temperature. Later, in 1884, Boltzmann derived the same result from the perspective of thermodynamics, and therefore the result is now called the Stefan-Boltzmann law,

$$\frac{P}{A} = \varepsilon\sigma T^4 \tag{1}$$

where P is the radiated power, A is radiating area, ε is the emissivity of the radiator ($=1$ for ideal radiator[1]), σ is Stefan-Boltzmann constant ($=5.67\times10^{-8}$ $\mathrm{Js^{-1}m^{-2}K^{-4}}$), and T is the temperature of the radiator. The amount of radiated energy in a given wavelength range depends on the temperature, and their relations were first discovered by Planck in 1900, which is usually marked as the origin of quantum theory.

2. Heat engine

A heat engine is a device that does work by extracting thermal energy from a hot reservoir and exhausting thermal energy to a cold reservoir. In this experiment, the heat engine consists of air inside a cylinder which expands when the attached can is immersed in hot water. The expanding air pushes on a piston and does work by lifting a weight. The

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[1] The ideal radiator is so-called "blackbody."

heat engine cycle is completed by immersing the can in cold water, returning the air pressure and volume to the starting values as Fig. 1 shows.

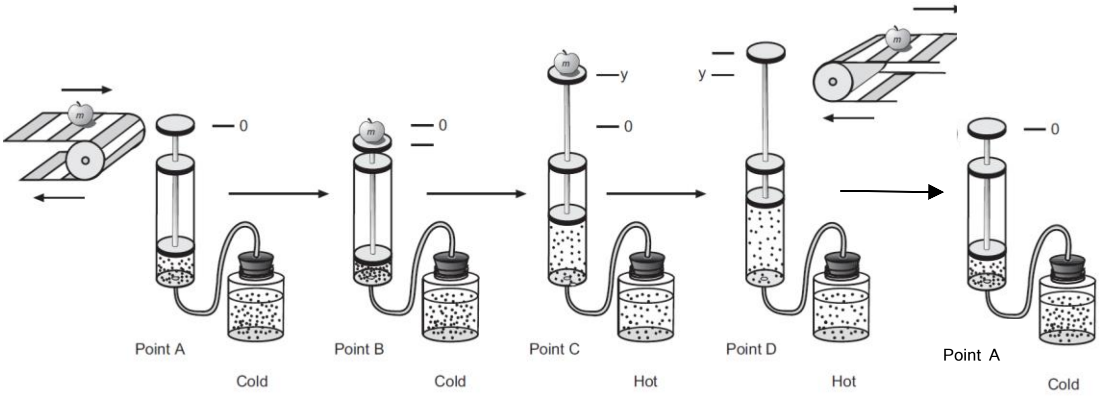


Fig. 1: Schematic for the process of a heat engine

At the beginning of the cycle, the air is held at a constant temperature while a weight is placed on top of the piston. Work is done on the gas and heat is exhausted to the cold reservior. The internal energy of the gas does not change since the temperature does not change. According to the first law of thermodynamics, the change in the internal energy is

$$\Delta U = Q - W = nC_V \Delta T = 0 \tag{2}$$

where Q is the heat added to the gas, W is the work done by the gas, and C_V is the molar heat capacity for constant volume.

In the second part of the cycle, heat is added to the gas, causing the gas to expand, pushing the piston up, doing work by lifting the weight. This process takes place at constant pressure since the piston is free to move. For an isobaric process, the heat added to gas is

$$Q_P = nC_P \Delta T \tag{3}$$

where n is the number of moles of gas in the container, C_P is the molar heat capacity for constant pressure, and ΔT is the change in temperature. The work done by the gas can again be found by the first law of thermodynamics. For air (mostly diatomic molecules),

$$C_V = \frac{5}{2}R \text{ and } C_P = \frac{7}{2}R \tag{4}$$

In the third part of the cycle, the weight is lifted off the piston while the gas is held at a hotter temperature. Heat is added to the gas and the gas expands, doing work. During this isothermal process, the work done by the gas is

$$W = nRT \ln\left(\frac{V_f}{V_i}\right) \tag{5}$$

where V_i is the initial volume at the beginning of the isothermal process and V_f is the final volume at the end of the isothermal process. Since the change in internal energy is zero for an isothermal process, the first law of thermodynamics shows that the heat added to the gas is equal to the work done by the gas: $\Delta U = Q - W = 0$.

In the final part of the cycle, heat is exhausted from the gas to the cold reservoir, returning the piston to its original position. This process is isobaric and the same equations apply as in the second part of the cycle.

The efficieny of a heat engine is defined as

$$e = \frac{W}{Q_H} \times 100\% \tag{6}$$

where W is the work done by the heat engine on its environment and Q_H is the heat extracted from the hot reservior. The theoretical maximum efficiency of a heat engine depends only on the temperature of the hot reservoir, $\mathrm{T_H}$, and the temperature of the cold reservior, $\mathrm{T_C}$. The maximum efficiency is given by

$$e_{\max} = \left(1 - \frac{\mathrm{T_C}}{\mathrm{T_H}}\right) \times 100\% \tag{7}$$

3. Isentropic Process

In thermodynamics, an isentropic process is an idealized thermodynamic process that is both adiabatic and reversible. Consider the ideal gas of one mole in isentropic expansion. Since no heat exchanges, the first law of thermodynamics becomes

$$\Delta U = Q - W = -W \tag{8}$$

If the system undergoes a very small change dV in volume, and the temperature change dT, accordingly, the change in the internal energy is

$$dU = -dW \Rightarrow C_V dT = -PdV \tag{9}$$

Recall the ideal gas equation, one obtains

$$\frac{C_V}{R}\left(PdV + VdP\right) = -PdV \Rightarrow C_P PdV + C_V VdP = 0 \tag{10}$$

Therefore, for an isentropic process, with γ defined as the heat capacity ratio, one has

$$\frac{C_P}{C_V}\frac{dV}{V} + \frac{dP}{P} = \gamma\frac{dV}{V} + \frac{dP}{P} = 0 \Rightarrow PV^{\gamma} = \text{Const.} \Rightarrow \mathrm{T}V^{\gamma-1} = \text{Const.} \tag{11}$$

C. Apparatus

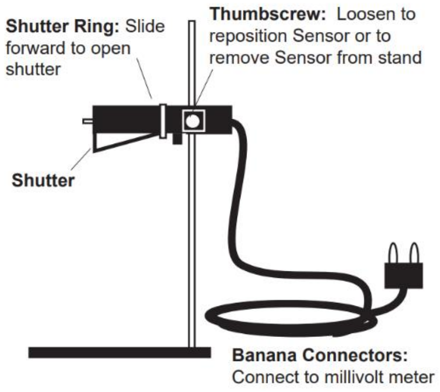


Fig. 2: Radiation Sensor. The radiation sensor measures the intensities of incident thermal radiation. The sensing element, a miniature thermopile, produces a voltage proportional to the intensity of the radiation.

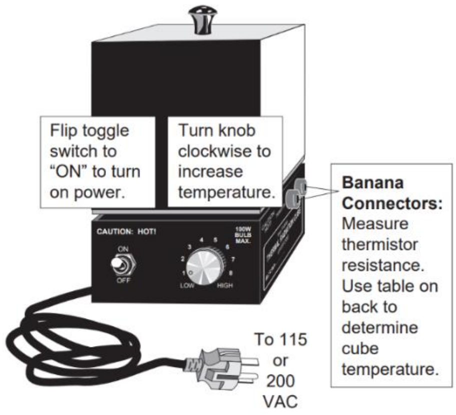


Fig. 3: Leslie's Cube (Radiation Cube). This cube provides four different radiating surfaces that can be heated from room temperature to approximately $120\ ^\circ\mathrm{C}$. The cube is heated by a 100 watt light bulb.

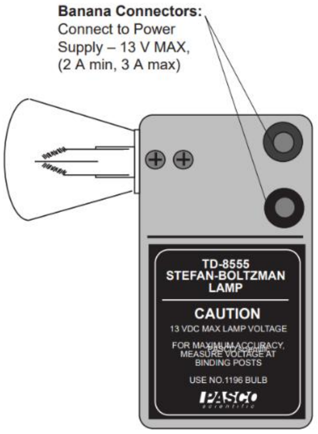


Fig. 4: Stefan-Boltzmann Lamp. It's a high temperature source of thermal radiation. The filament temperature is determined by measuring the voltage and current into the lamp.

Fig. 5: Heat Engine Apparatus. It's designed with two pressure ports with quick-connect fittings for connecting to the air chamber tubing.

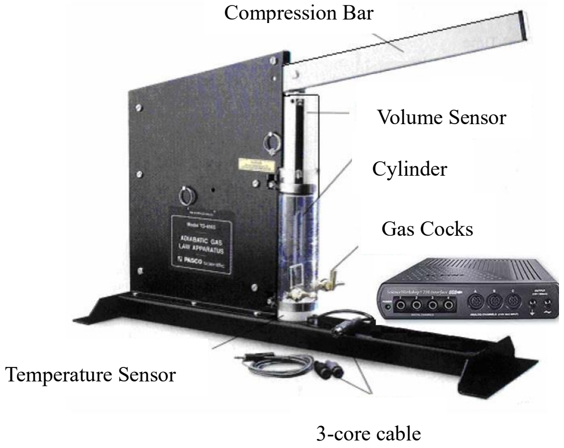


Fig. 6: Adiabatic Gas Law Apparatus. Pressure, temperature and volume are measured during the experiment. Compress the air in the cylinder by pressing down on the piston. An increase in temperature and pressure should be observed on the graphs in Logger Pro, while there is a decrease in volume. When the piston is raised to the upper limit, the Temperature and Pressure will decrease, while the volume increases.

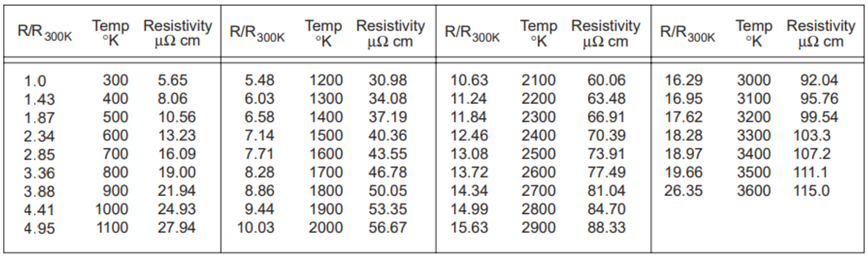
D. Procedures

1. Pre-lab assignments (hand in before the lab)

(1) Make a flowchart of this experiment and answer the questions

(i) During the experiment, you are going to use Stefan-Boltzmann lamp to verify the Stefan-Boltzmann Law (See exp. 1-2). Table 1 shows the relations between Temperature and Resistivity for Tungsten, where R/R_{300K} is the relative resistance compared with the resistance at room temperature. Given that the voltage difference across the lamp and the current passing through it at room temperature are $0.19\ \mathrm{V}$ and $0.426\ \mathrm{A}$, use the method of interpolation to find the temperature of the lamp when the voltage difference and the current passing through the lamp are $(7,\ 1.972)$, $(10,\ 2.386)$, and $(12,\ 2.635)$.
Moreover, if the room temperature $\mathrm{T_{room}}$ is now $10°\mathrm{C}$, how do you obtain the correct resistence of the filament at $300\ \mathrm{K}$?

Table 1: Temperature and Resistivity for Tungsten



R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm	R/R_{300K}	Temp °K	Resistivity μΩ cm
1.0	300	5.65	5.48	1200	30.98	10.63	2100	60.06	16.29	3000	92.04
1.43	400	8.06	6.03	1300	34.08	11.24	2200	63.48	16.95	3100	95.76
1.87	500	10.56	6.58	1400	37.19	11.84	2300	66.91	17.62	3200	99.54
2.34	600	13.23	7.14	1500	40.36	12.46	2400	70.39	18.28	3300	103.3
2.85	700	16.09	7.71	1600	43.55	13.08	2500	73.91	18.97	3400	107.2
3.36	800	19.00	8.28	1700	46.78	13.72	2600	77.49	19.66	3500	111.1
3.88	900	21.94	8.86	1800	50.05	14.34	2700	81.04	26.35	3600	115.0
4.41	1000	24.93	9.44	1900	53.35	14.99	2800	84.70			
4.95	1100	27.94	10.03	2000	56.67	15.63	2900	88.33			

(ii) Consider a heat engine cycle in exp. 2.

(a) Draw a graph of P versus V, and put arrows on the graph to identify each process. Label the temepratures of the reservoirs by $\mathrm{T_H}$ and $\mathrm{T_L}$.

(b) Show that the heat added to the gas during the process $B \to C$ is

$$Q_{B\to C} = \frac{7}{2}\frac{P_D V_D}{T_H}(T_H - T_L) \tag{12}$$

where P_D and V_D represent the pressure and the volume at point D.

(c) The efficiency of the heat engine is defined in eq(6). Explain how you obtain Q_H and W if the P-V diagram is given.

(iii) What are the theoretical γ values of air, carbon dioxide, and helium? Explain how you obtain the value. Also, search for the real values on the internet.

(iv) Consider an adiabatic compression process in exp. 3. If the gas initially has pressure P_1 and volume V_1 and after the compression, it has pressure P_2 and volume V_2, show that the work done to compress the gas under this adiabatic compression process is

$$W = \frac{P_1 V_1^{\gamma}}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) \tag{13}$$

2. In-lab activities

(1) Thermal radiation

1-1 Radiation rates from different surfaces

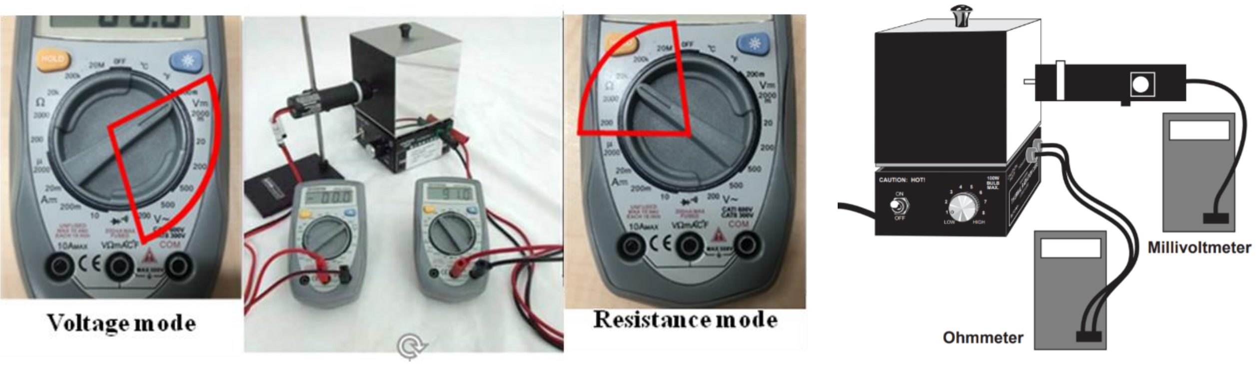
(i) Switch the modes of two multimeters and connect them as shown in Fig. 7.

(ii) Turn on the Thermal Radiation Cube and set the power switch to “HIGH”. When the ohmmeter reading gets down to about $40\ \text{k}\Omega$, reset the power switch to 5.0.

(iii) When the cube reaches thermal equilibrium, where the ohmmeter reading fluctuates around a relatively fixed value, use the Radiation Sensor to measure the radiation emitted from each of the four surfaces of the cube, and record your measurements. **(Note: place the Sensor so that the posts on its end are in contact with the cube surface, which ensures the same distance of the measurement for all surfaces.)**

(iv) Measure and record the resistance of the thermistor. Use Table 2 to determine the corresponding temperature.

(v) Increase the power switch setting, first to 7.0, then to “HIGH.” At each setting, repeat steps (iii) and (iv) and record your results.


Voltage mode

Resistance mode

Fig. 7: Experimental setup for observing radiation rates from different surfaces

1-2 Stefan-Boltzmann law

(i) **Before turning on the lamp**, measure the room temperature T_{ref} in Kelvin,

and the resistance R_{ref} of the filament of the lamp at room temperature.

(ii) Set up the equipment as shown in Fig. 8. The voltmeter should be connected directly to the binding posts of the lamp. The sensor should be at the same height as the filament, with the front face of the Sensor approximately $6\ \text{cm}$ away from the filament. **The entrance angle of the thermopile should include no close objects other than the lamp.**

(iii) Turn on the power supply. Set the voltage $V = 1, 2, 3, \ldots\ldots, 11, 12\ \text{V}$. At each voltage setting, record the ammeter reading and voltmeter reading.

(iv) Calculate the resistance R of the filament and determine the temperature T of the lamp filament at each voltage setting. **(Hint: Pre-lab assigment)**

(v) Construct graphs of $\text{Rad vs. T, Rad vs. T}^4$, and $\log(\text{Rad})$ vs. $\log(\text{T})$.

NOTE: The voltage into the lamp should NEVER exceed 13 V, and you should make each reading quickly. Between readings, place both sheets of insulating foam between the lamp and the Sensor, with the silvered surface facing the lamp, so that the temperature of the Sensor stays relatively constant.

Table 2: Resistance versus Temperature for the Thermal Radiation Cube

Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)	Therm. Res. (Ω)	Temp. (°C)
207,850	10	66,356	34	24,415	58	10,110	82	4,615.1	106	2,281.0	130
197,560	11	63,480	35	23,483	59	9,767.2	83	4,475.0	107	2,218.3	131
187,840	12	60,743	36	22,590	60	9,437.7	84	4,339.7	108	2,157.6	132
178,650	13	58,138	37	21,736	61	9,120.8	85	4,209.1	109	2,098.7	133
169,950	14	55,658	38	20,919	62	8,816.0	86	4,082.9	110	2,041.7	134
161,730	15	53,297	39	20,136	63	8,522.7	87	3,961.1	111	1,986.4	135
153,950	16	51,048	40	19,386	64	8,240.6	88	3,843.4	112	1,932.8	136
146,580	17	48,905	41	18,668	65	7,969.1	89	3,729.7	113	1,880.9	137
139,610	18	46,863	42	17,980	66	7,707.7	90	3,619.8	114	1,830.5	138
133,000	19	44,917	43	17,321	67	7,456.2	91	3,513.6	115	1,781.7	139
126,740	20	43,062	44	16,689	68	7,214.0	92	3,411.0	116	1,734.3	140
120,810	21	41,292	45	16,083	69	6,980.6	93	3,311.8	117	1,688.4	141
115,190	22	39,605	46	15,502	70	6,755.9	94	3,215.8	118	1,643.9	142
109,850	23	37,995	47	14,945	71	6,539.4	95	3,123.0	119	1,600.6	143
104,800	24	36,458	48	14,410	72	6,330.8	96	3,033.3	120	1,558.7	144
100,000	25	34,991	49	13,897	73	6,129.8	97	2,946.5	121	1,518.0	145
95,447	26	33,591	50	13,405	74	5,936.1	98	2,862.5	122	1,478.6	146
91,126	27	32,253	51	12,932	75	5,749.3	99	2,781.3	123	1,440.2	147
87,022	28	30,976	52	12,479	76	5,569.3	100	2,702.7	124	1,403.0	148
83,124	29	29,756	53	12,043	77	5,395.6	101	2,626.6	125	1,366.9	149
79,422	30	28,590	54	11,625	78	5,228.1	102	2,553.0	126	1,331.9	150
75,903	31	27,475	55	11,223	79	5,066.6	103	2,481.7	127		
72,560	32	26,409	56	10,837	80	4,910.7	104	2,412.6	128		
69,380	33	25,390	57	10,467	81	4,760.3	105	2,345.8	129		

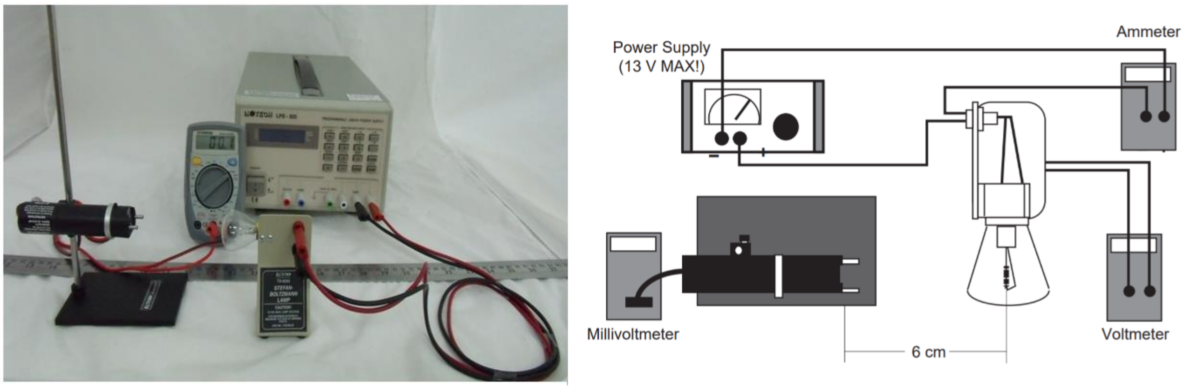


Fig. 8: Experimental setup for verifying the Stefan-Boltzmann law

(2) Heat engine

(i) Set up the experiment as shown in Fig. 9.

(a) The Heat engine should be oriented with the piston end up and the Heat Engine should be positioned close to the bottom of the rod stand.

(b) Thread one end of a string through the hole in the top of the piston platform and tie that end of the string to the shaft of the piston under the piston platform. Pass the other end of the string over the medium step of the pulley and attach a 35-gram hanging mass.

(c) Position the piston 25~30 mL from the bottom of the cylinder, attach the tube from the can to one port on the Heat Engine and close the shut-off valve on the tubing from the unused port. (See Fig. 5)

(d) Put hot water (about 80°C) into one of the plastic containers (about 90% full), and ice water in the other one. (Note: Containers are labelled as shown.)

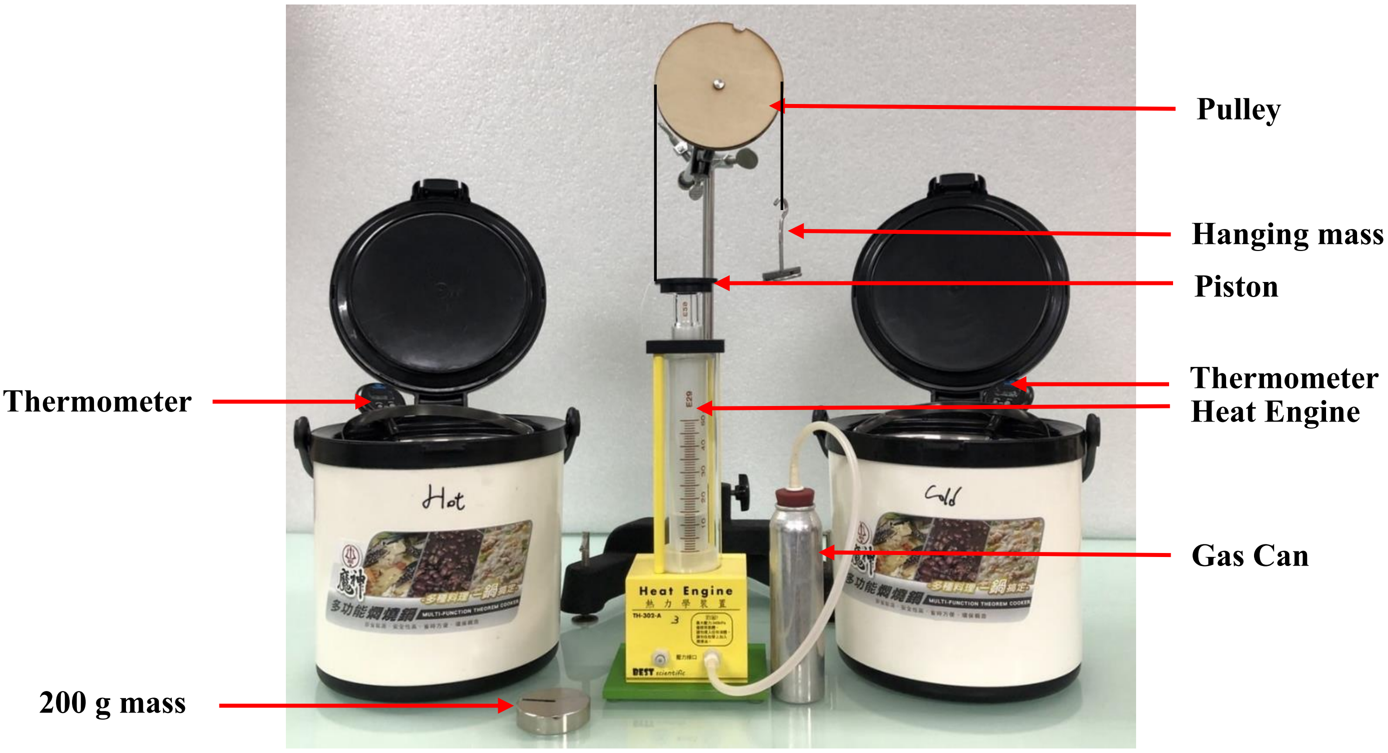


Fig. 9: Experimental setup of heat engine

(ii) Perform the following cycle without hesitating between steps. **You may want to practice a few times before recording a data run**. Start with the can in the cold water. This starting point will be called point A. Record the height of the bottom of the piston. **(See Fig. 1)**

(a) $A \rightarrow B$: Place the 200 g mass on the platform.

(b) $B \rightarrow C$: Move the can from the cold bath to the hot bath.

(c) $C \rightarrow D$: Remove the 200 g mass from the platform.

(d) $D \rightarrow A$: Move the can from the hot bath to the cold bath.

(iii) Record the pressure[2] and the volume[3] of A, B, C, and D. Label the temperatures at these points and put arrows on the cycle to show the direction of the process. Also, identify the types of processes (i.e., isothermal, etc.) and the actual physical performance (put mass on, put in a hot bath, etc.) for the four processes.

(iv) Calculate the work done by the 200 g mass, and compare with the area formed by the four corners in the P-V diagram.

(v) Calculate the efficiency of the Heat Engine and compare with the theoretical maximum efficiency

(vi) Change the temeprature difference between hot and cold water and re-do the experiment. Discuss the effect of the temeprature difference in the post-lab report.

(3) Isentropic compression

(i) Set up the experiment as shown in Fig. 10.

(ii) Three gas are given to be compressed: Air, Carbon dioxide, and Helium. Do the following steps for each gas.

(iii) To measure temperature, pressure, and volume by LabQuest mini and Loggerpro:

(a) Click "Collect" in Loggerpro,

(b) Press down the compression bar to the bottom in a short time, and keep still.

(c) Click "Stop" in Loggerpro.

(d) Export data in "csv" file format.

(iv) Compare the final pressure and temeprature with the values predicted by the theory (eq 11). Calculate the work done to the gas under the process.

(v) Instead of keeping still after pressing down the bar to the bottom, release the bar immediately to collect the data for these two processes. You will observe the data form a closed curve in lnP-lnV diagram; however, for an isentropic process, it should be just a straight line. What's the meaning of the area formed by the curve? Explain in your post-lab report.

(Question: What is the difference if you don't conduct the experiment in a short time?)

Fig. 10: Experimental setup of Isentropic compression

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[2] Asuume the original pressure is equal to atmospheric pressure (measured by Phyphox), and the pressure caused by the weight can be directly calculated given the cross area of the cylinder being $A = 5\ \mathrm{cm}^2$.

[3] Read directly from the cylinder scales.

3. Post-lab report
   (1) Recopy and organize your data from the in-lab tables in a neat and more readable form.
   (2) Analyze the data you obtained in the lab and answer the given questions
   (3) Compare the results with the theory, and discuss the uncertainties that occur in the experiments, and how they influence the experiments. (Quantitatively, if possible.)

E. Questions

1. Approximate Planetary Temperatures by Stefan-Boltzmann Law

In the solar system, the surface temperatures of the planets are basically due to the heat provided by the Sun. Assuming the fraction of the power emitted by the Sun that is absorbed by the planet at the distance r from the Sun can be set equal to the power emitted by the planet, (a) find surface temperature $T(r)$ of the planet at a distance r from the Sun in terms of $r, \mathrm{T_{Sun}}$, and $\mathrm{R_{Sun}}$. (b) Given that the temperature of the Sun $\mathrm{T_{Sun}} = 5800$ K and its radius $R_{Sun} = 6.95\times10^{8}$ m, estimate the approximate surface temperatures of the planets in the solar system. (Their distances from the Sun are given in the table below.) (c) Compare the results with the reported values as shown in the Table. Explain what you find.

Planet	Distance from Sun (AU)	Reported Temperature (K)
Mercury	0.39	452
Venus	0.723	726
Earth	1	285
Mars	1.52	230
Jupiter	5.2	120
Saturn	9.539	88
Uranus	19.18	59
Neptune	30.06	48

2. Human Respiration as a Heat Engine

Thinking of a lung as a rubber ballon, it collapses to nearly zero volume at atmospheric pressure, which corresponds to the internal energy $U = 5PV/2 = 0$ with P the gauge pressure. During respiration, both the number of moles and the temperature of the air in the lungs change. The empirical fit obtained by the real data satisfies

$$V = A\left(1-e^{-kP}\right)$$

with values $\mathrm{A_I}$ and $\mathrm{k_I}$ during inhalation, and $\mathrm{A_E}$ and $\mathrm{k_E}$ during exhalation being

$$\begin{cases} A_I = 3.57\ \mathrm{L},\ \ k_I = 0.964\times10^{-3}\ \mathrm{Pa^{-1}} \\ A_E = 3.35\ \mathrm{L},\ \ k_E = 1.644\times10^{-3}\ \mathrm{Pa^{-1}} \end{cases}$$

(a) Plot a pressure-volume respiration cycle by the given information.

(b) Calculate the net work done by a healthy lung for a cycle of respiration.

(c) Estimate the net work done by your lung in a day.

F. References

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Instruction Manual and Experiment Guide for the PASCO scientific Model TD-8553/8554A/8555

Instruction Manual and Experiment Guide for the PASCO scientific Model TD-8565